Functions Review: Graphs, Transformations, and Rate of Change

Pal4Pal Summer PreCalc Week 1 Session 1

06.22.2026

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Contents

1 Curriculum Overview 3
2 Function Basics 5
3 Graphing Functions 16
4 Rate of Change 21
5 Rate of Change Example 23
6 Composition 26
7 Inverse Functions 28
8 Review 35
9 Homework 38

1 Curriculum Overview

Contents (iii)

1. Function Review
2. Introduction to Trigonometry
3. Trigonometric Identities
4. Applications to Geometry
5. Parameterization and Trigonometric Coordinate Systems
6. Basics of Complex Numbers
7. Trigonometry and Complex Numbers
8. Vectors in Two Dimensions
9. Matrices in Two Dimensions

2 Function Basics

Contents (v)

Domain: consists of all the values we are able to input to the function and get an output

Range: consists of all the values that can come out of the function

Example:

$$y = 2x + 3$$

Domain: All Real Numbers

Range: All Real Numbers

Example:

$$\frac{1}{x-3}$$

Domain: All real numbers except 3

Range: All real numbers except 0

Contents (vi)

Real-valued Function (aka Real Function): a function that can only output real numbers

A lot of the times, we use **interval notation** to express the domain and the range of a function.

We use "(" and ")" to indicate that the boundary values are not included;

"[" and "]" to indicate that the values are included as boundary values.

Example:

$$x \in (3, 5) \rightarrow 3 < x < 5$$

$$y \in [0, 1.223] \rightarrow 0 \leq y \leq 1.223$$

$$x \in (3, \infty) \text{ or } x \in (3, +\infty) \rightarrow x > 3$$

p.s. To write all real numbers in interval notation, we can write $(-\infty, +\infty)$ or $(-\infty, \infty)$ or $x \in \mathbb{R}$

Contents (vii)

Quick Question!!!

What does $x \in (7, 10.5]$ represent?

$A. 7 \leq x < 10.5$

$B. 7 < x \leq 10.5$

$C. 7 \leq x \leq 10.5$

$D. 7 < x < 10.5$

E. What is this I don't get it 😭🙏🙏🙏

Contents (viii)

Problem 1.1: Let $f(x) = 2x^2 - 3$.

(a) Find $f(2)$.

(b) Find all values of t such that $f(t) = 47$.

(c) Find $f(3x + 1)$.

Contents (ix)

Problem 1.2: A function can be defined to accept multiple inputs. For example, let

$$g(a, b, c) = 3a - 2b + 7c^2.$$

(a) Evaluate $g(3, -5, -1)$.

(b) Find b if $g(b, 2, -1) = 21$.

Contents (x)

Problem 1.3: Suppose $t(x) = ax^4 + bx^2 + x + 5$, where a and b are constants. In this problem, we find $t(4)$ if $t(-4) = 3$.

(a) Use $t(-4) = 3$ to write an equation in terms of a and b.

(b) Express $t(4)$ in terms of a and b.

(c) Find $t(4)$.

Contents (xi)

Problem 1.4: Find the domain and range of each of the following functions:

(a) $f(x) = 2x - 3$

(b) $f(x) = \sqrt{-2x + 7}$

(c) $g(t) = \dfrac{2t}{t - 1}$

(d) $h(x) = 9x^2 + 4$

Contents (xii)

Problem 1.5: Suppose $p(x) = 2\sqrt{x} + 3$, but that $p(x)$ is only defined for $4 \leq x \leq 9$. In other words, the domain of p is all real numbers from 4 to 9.

(a) Is there a value of x for which $p(x) = 5$?

(b) What is the range of p?

Contents (xiii)

Problem 1.6: Find the domain of each of the following functions:

(a) $f(x) = \dfrac{\sqrt{2x-5}}{x-3}$

(b) $g(t) = \dfrac{2t-4}{\frac{1}{t} - \frac{1}{3t-4}}$

Contents (xiv)

Problem 1.7: Notice that $\frac{x^2 - x}{x - 1} = \frac{(x - 1)(x)}{x - 1} = x$. Are $f(x) = \frac{x^2 - x}{x - 1}$ and $g(x) = x$ the same function?

3 Graphing Functions

Contents (xvi)

The ***x*-intercepts** of a graph are the points where the graph crosses the *x*-axis (the point(s) in the graph where y=0)

The ***y*-intercepts** of a graph are the points where the graph crosses the *y*-axis (the point in the graph where x=0)

Example:

$$y = x^2 - 4$$

x-intercepts:

$$0 = x^2 - 4$$

$$4 = x^2$$

$$x = -2 \text{ or } x = 2$$

The *x*-intercepts of the function are: (-2,0) and (2,0)

y-intercepts:

$$y = (0)^2 - 4$$

$$y = -4$$

The *y*-intercept of the function is: (0,-4)

Contents (xvii)

Problem 1.8: For each of the functions below, graph the function and find the x-intercepts and the y-intercepts of the graph.

(a) $f(x) = (x-6)^2 - 4$

(b) $f(x) = 4$

(c) $f(x) = \sqrt{x+5} - 3$

(d) $f(x) = |x+2| + 1$

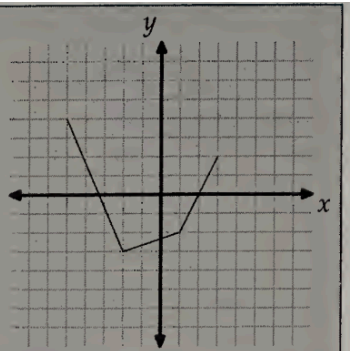
Contents (xviii)

Problem 1.11: At right is the graph of $y = f(x)$.

(a) Explain why the graph of $y = f(x) + 2$ is a 2-unit upward shift of the graph of $y = f(x)$.

(b) Explain why the graph of $y = f(x + 2)$ is a 2-unit leftward shift of the graph of $y = f(x)$.

(c) Find the domain and range of h if $h(x) = f(x + 3) - 4$.



Contents (xix)

Problem 1.12: Let $f(x)$ be the function graphed in the previous problem.

(a) Find the graph of $y = -f(x)$.

(b) Find the graph of $y = f(-x)$.

(c) Find the graph of $y = 2f(x)$.

(d) Find the graph of $y = f(2x)$.

4 Rate of Change

Contents (xxi)

Rate of Change measures how much the output changes compared to how much the input changes.

For two points (x_1, y_1) and (x_2, y_2):

$$\text{Rate of Change} = \frac{\text{change in output}}{\text{change in input}} = \frac{y_2 - y_1}{x_2 - x_1}$$

For a function $f(x)$ on the interval $[a, b]$:

$$\text{Average Rate of Change} = \frac{f(b) - f(a)}{b - a}$$

This is the same idea as **slope**.

If the rate of change is positive, the function is increasing on average.

If the rate of change is negative, the function is decreasing on average.

If the rate of change is zero, the output did not change overall.

5 Rate of Change Example

Contents (xxiii)

Suppose $f(x) = x^2$.

Find the average rate of change from $x = 1$ to $x = 4$.

$$\frac{f(4) - f(1)}{4 - 1} = \frac{16 - 1}{3} = \frac{15}{3} = 5$$

So the average rate of change is 5.

Meaning: from $x = 1$ to $x = 4$, the function output increases by about 5 units for every 1 unit increase in x.

Graphically, this is the slope of the line through the two points $(1, f(1))$ and $(4, f(4))$, or $(1, 1)$ and $(4, 16)$.

Quick Question!!!

Find the average rate of change of $g(x) = 2x + 3$ from $x = 0$ to $x = 5$.

A.2

B.3

Contents (xxiv)

C.5

D.13

E. What is rate of change again 😭

6 Composition

Contents (xxvi)

Composition is when we take the output from one function and put it into another function.

The expression *f(g(x))* or $f \circ g$ means that we put our input into function *g*, and then take the output, *g(x)*, into function *f(x)*.

IMPORTANT!!! f(g(x)) $\neq$ g(f(x))

Example:

Suppose *f(x)* = 3x-7 and *g(x)* = $2x^2$.

(a) find *f(g(3))*

(b) find *g(f(3))*

7 Inverse Functions

Contents (xxviii)

A function that "undoes" the work of a function f is called the **inverse function** of f.

Definition: Function g is the inverse of function f if

- $g(f(x)) = x$ for all values of x in the domain of f

AND

- $f(g(x)) = x$ for all values of x in the domain of g

However, note that not every function has an inverse! In a function, each input value (x) can only have one output value(y), so if a function has multiple inputs value that has the same output value, the function does not have an inverse function.

Contents (xxix)

Problem 1.18: In this problem we find the inverse of the function $f(x) = 3x - 5$.

(a) Let g be the inverse of f, so that we have $f(g(x)) = x$ and $g(f(x)) = x$. Which of these equations is easier to use to find $g(x)$?

(b) Use the definition of f to solve the equation $f(y) = x$ for y.

(c) What is $g(x)$?

(d) Check your answer by confirming that $g(f(x)) = x$.

Contents (xxx)

Problem 1.19: Let $f(x) = \dfrac{-5-x}{3-2x}$.

(a) Find $f^{-1}(7)$, *without finding* $f^{-1}(x)$.

(b) Find $f^{-1}(x)$.

Contents (xxxi)

Problem 1.20:

(a) Does the function $g(x) = 2x^2 - 9$ have an inverse? If it does, find it. If it doesn't, explain why it doesn't.

(b) Does the function $h(x) = 3\sqrt{2x + 1}$ have an inverse? If it does, find it. If it doesn't, explain why it doesn't. (Be careful on this part!)

Contents (xxxii)

Problem 1.21:

(a) Explain how to determine from the graph of a function whether or not the function has an inverse.

(b) How are the graph of a function and the graph of its inverse related?

Contents (xxxiii)

Problem 1.22: Suppose functions f and g both have inverses. Show that if $h = f \circ g$, then the inverse of h is $g^{-1} \circ f^{-1}$.

8 Review

Contents (xxxv)

Domain

all the values we are able to input to the function and get an output

Range

all the values that can come out of the function

Real-valued Function (aka Real Function)

a function that can only output real numbers

Interval Notation

used to represent a continuous range of real numbers along a number line

***x*-intercept**

the points where the graph crosses the x-axis (the point(s) in the graph where y=0)

Contents (xxxvi)

y-intercept

the points where the graph crosses the y-axis (the point(s) in the graph where x=0)

Composition

when we take the output from one function and put it into another function

Inverse Function

A function that “undoes” the work of another function

9 Homework

Contents (xxxviii)

P23 question 51-73

- multiples of 3
- only need to list the intercepts

Read P56-64

P68 15-21odd, 39, 41, 49-59 odd, 65, 69, 71-73, 77, 78, 83-85, 98

P88-90 6-10, 11-19 odd