

Intro to Trigonometric Functions: Radians, Unit Circle, and Exact Values

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1 Session 1 Functions Review

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Before trig, remember the main idea from functions:

Function

each input has exactly one output

Domain

allowed input values

Range

possible output values

Graph

visual representation of input-output behavior

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Inverse

a function that reverses another function, when it exists

Today, the inputs will be **angles**, and the outputs will be numbers like $\sin \theta$, $\cos \theta$, and $\tan \theta$.

Quick Question!!!

If $\theta = 30^\circ$, what kind of object is θ ?

- A. an input
- B. an output
- C. a graph
- D. a range

2 Why Trigonometry?

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Trigonometry studies relationships between angles, side lengths, and circular motion.

Two common ways to build trig:

1. **Right-triangle approach:** ratios of side lengths
2. **Unit-circle approach:** coordinates of a point moving around a circle

This class will use both intuition from triangles and the unit-circle definition used in precalculus.

Main idea:

angle $\rightarrow$ point on a circle $\rightarrow$ coordinates $\rightarrow$ trig values

3 Geometric Introduction of Trigonometry

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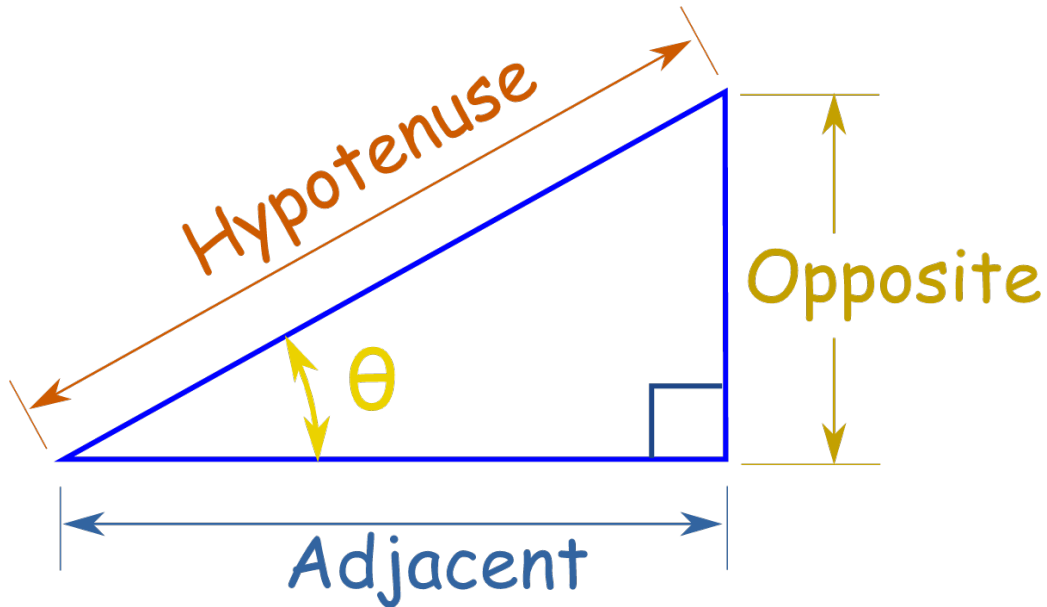
For a right triangle with angle θ :

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

Memory tool: **SOH-CAH-TOA**



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Example

If the opposite side is 3, adjacent side is 4, and hypotenuse is 5, then

$$\sin \theta = \frac{3}{5}, \quad \cos \theta = \frac{4}{5}, \quad \tan \theta = \frac{3}{4}$$

4 Special Triangles

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The exact values we use most often come from two triangles.

Triangle	Side ratio	Useful angles
45° - 45° - 90°	$1 : 1 : \sqrt{2}$	$45^\circ = \frac{\pi}{4}$
30° - 60° - 90°	$1 : \sqrt{3} : 2$	$30^\circ = \frac{\pi}{6}, 60^\circ = \frac{\pi}{3}$

5 Special Triangle Examples

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For a 45° - 45° - 90° triangle:

$$\sin 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan 45^\circ = \frac{1}{1} = 1$$

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For a 30° - 60° - 90° triangle:

$$\sin 30^\circ = \frac{1}{2}, \quad \cos 30^\circ = \frac{\sqrt{3}}{2}, \quad \tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2}, \quad \cos 60^\circ = \frac{1}{2}, \quad \tan 60^\circ = \sqrt{3}$$

6 Angles and Radians

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Definition:

$$\theta = \frac{s}{r}$$

where s is arc length and r is radius.

Important conversions:

$$180^\circ = \pi \text{ radians}$$

$$360^\circ = 2\pi \text{ radians}$$

$$\text{degrees} \rightarrow \text{radians} : \theta \cdot \frac{\pi}{180}$$

$$\text{radians} \rightarrow \text{degrees} : \theta \cdot \frac{180}{\pi}$$

7 Radian Practice

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Quick Question!!!

Convert each angle into radian.

1. $30^\circ = ?$
2. $45^\circ = ?$
3. $60^\circ = ?$
4. $90^\circ = ?$
5. $180^\circ = ?$
6. $270^\circ = ?$

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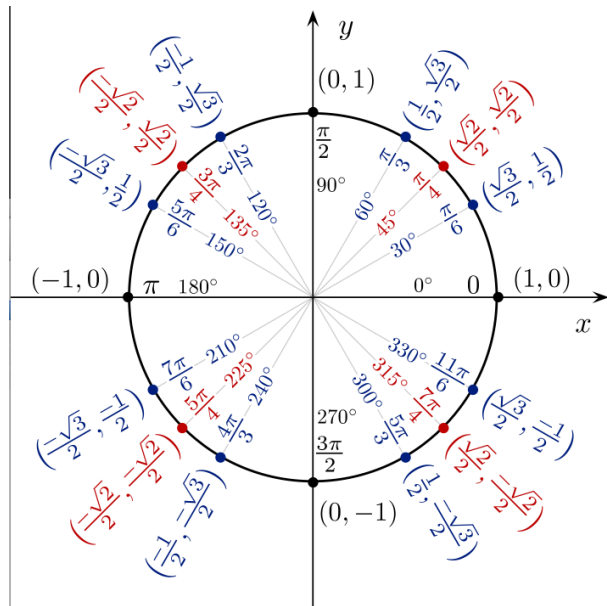
Answers:

$$30^\circ = \frac{\pi}{6}, \quad 45^\circ = \frac{\pi}{4}, \quad 60^\circ = \frac{\pi}{3}$$

$$90^\circ = \frac{\pi}{2}, \quad 180^\circ = \pi, \quad 270^\circ = 3\frac{\pi}{2}$$

8 Unit Circle

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The **unit circle** is the circle centered at the origin with radius 1.

$$x^2 + y^2 = 1$$

A point $P = (x, y)$ on the unit circle corresponds to an angle θ .

Key definition:

$$\cos \theta = x$$

$$\sin \theta = y$$

$$\tan \theta = \frac{y}{x} \quad (x \neq 0)$$

So cosine is horizontal position, sine is vertical position, and tangent is slope from the origin.

9 Angles in Standard Position

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An angle is in **standard position** if:

1. its vertex is at the origin
2. its initial side lies on the positive x -axis
3. positive angles rotate counterclockwise
4. negative angles rotate clockwise

Quadrants matter because signs come from coordinates.

Quadrant	Signs of (x, y)	Signs of $(\cos \theta, \sin \theta)$
I	$(+, +)$	$(+, +)$
II	$(-, +)$	$(-, +)$
III	$(-, -)$	$(-, -)$
IV	$(+, -)$	$(+, -)$

10 Quadrantal Angles

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At multiples of 90° , the point on the unit circle lies on an axis.

Angle	Radians	$P = (x, y)$	$\cos \theta$	$\sin \theta$
0°	0	$(1, 0)$	1	0
90°	$\frac{\pi}{2}$	$(0, 1)$	0	1
180°	π	$(-1, 0)$	-1	0
270°	$3\frac{\pi}{2}$	$(0, -1)$	0	-1
360°	2π	$(1, 0)$	1	0

11 Exact Values on the Unit Circle

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Angle	Radians	$\cos \theta$	$\sin \theta$
30°	$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
60°	$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$

Pattern: as the angle increases from 0° to 90° , cosine decreases and sine increases.

12 Trigonometric Functions

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The six trig functions are defined from $P = (x, y)$ on the unit circle.

Function	Definition	Undefined when...
$\sin \theta$	y	never on unit circle
$\cos \theta$	x	never on unit circle
$\tan \theta$	$\frac{y}{x}$	$x = 0$
$\csc \theta$	$\frac{1}{y}$	$y = 0$
$\sec \theta$	$\frac{1}{x}$	$x = 0$
$\cot \theta$	$\frac{x}{y}$	$y = 0$

13 Signs by Quadrant

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Since $\cos \theta = x$ and $\sin \theta = y$, signs come from the quadrant.

Quadrant	$\sin \theta$	$\cos \theta$	$\tan \theta$
I	+	+	+
II	+	—	—
III	—	—	+
IV	—	+	—

Tangent is positive when sine and cosine have the same sign.

14 Reference Angles

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A **reference angle** is the acute angle between the terminal side and the x -axis.

Strategy for exact values:

1. Find the reference angle.
2. Use the special triangle value.
3. Choose the sign from the quadrant.

Example

Find $\sin(5\frac{\pi}{6})$, $\cos(5\frac{\pi}{6})$, and $\tan(5\frac{\pi}{6})$.

$5\frac{\pi}{6}$ is in Quadrant II and has reference angle $\frac{\pi}{6}$.

$$\sin(5\frac{\pi}{6}) = \frac{1}{2}, \quad \cos(5\frac{\pi}{6}) = -\frac{\sqrt{3}}{2}, \quad \tan(5\frac{\pi}{6}) = -\frac{\sqrt{3}}{3}$$

15 Practice Problems

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Find exact values. No calculators.

1. $\sin\left(\frac{\pi}{3}\right)$
2. $\cos\left(\frac{\pi}{4}\right)$
3. $\tan\left(\frac{\pi}{6}\right)$
4. $\sin\left(7\frac{\pi}{6}\right)$
5. $\cos\left(4\frac{\pi}{3}\right)$
6. $\tan\left(3\frac{\pi}{4}\right)$

Then explain each answer using either a special triangle or the unit circle.

16 Calculator Warning

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Your calculator has two angle modes:

1. **Degree mode:** use for inputs like 30° , 120° , -45°
2. **Radian mode:** use for inputs like $\frac{\pi}{6}$, $2\frac{\pi}{3}$, $-\frac{\pi}{4}$

Common mistake:

Typing $\sin(30)$ in radian mode does **not** mean $\sin(30^\circ)$.

Before using a calculator, always check the mode.

17 Review

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Trigonometry

study of angles, side ratios, and circular motion

Radian

angle measure defined by $\theta = \frac{s}{r}$

Unit Circle

circle $x^2 + y^2 = 1$

Cosine

$\cos \theta = x$

Sine

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$$\sin \theta = y$$

Tangent

$$\tan \theta = \frac{y}{x}$$

Special Angles

$$\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$$

Reference Angle

acute angle to the x -axis

Quadrant Signs

signs of trig values come

18 Homework

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Read Section 5.1: angles, radians, arc length, and circular motion.

Read Section 5.2: unit circle approach and exact values.

Do:

- Section 5.1: problems 1–8, 83–86
- Section 5.2: problems 9–19 odd, 33, 75, 79, 80
- Optional challenge: fill out a blank unit circle with degrees, radians, and coordinates.